

Practice Problems for Test #3

1. 1. $G \supset H$
 2. $G \supset I$ / $G \supset (H \cdot I)$

2. 1. $J \equiv K$
 2. $\sim K \equiv \sim L$ / $J \equiv L$

3. 1. M / $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$

4. Prove that ' $R \supset [(R \supset S) \supset S]$ ' is a logical truth.

5. 1. $D \supset E$
 2. $E \supset (F \cdot G)$
 3. $\sim F \vee \sim G$ / $\sim D$

6. 1. $(P \vee Q) \vee (\sim R \vee S)$
 2. $\sim Q \cdot (\sim S \cdot \sim P)$ / $\sim R$

7. 1. $A \supset B$
 2. $B \supset D$
 3. $D \supset A$
 4. $A \supset \sim D$ / $\sim A \cdot \sim D$

8. Prove that ' $A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$ ' is a logical truth

9. 1. $(A \vee B) \supset \sim(F \cdot D)$
 2. $\sim(A \cdot \sim D)$
 3. $\sim F \supset \sim(C \cdot D)$
 4. $C \vee A$ / $A \equiv \sim C$

10. 1. $(A \cdot B) \supset C$
 2. $(F \cdot D) \supset E$
 3. $A \cdot D$
 4. $B \vee F$ / $C \vee E$

11. 1. $\sim F \supset (G \cdot H)$
 2. $F \supset (I \vee J)$
 3. $\sim[G \cdot (K \cdot \sim J)]$
 4. $K \equiv (L \cdot \sim J)$ / $K \supset I$

12. 1. $(I \cdot E) \supset \sim F$
 2. $F \vee (G \cdot H)$
 3. $I \equiv E$ / $I \supset G$

13. Prove that ' $(P \cdot Q) \supset [(P \vee R) \cdot (Q \vee R)]$ ' is a logical truth.

14. Prove that $(A \supset B) \vee (\sim A \supset C)$ is a logical truth.

15. 1. $(T \supset W) \cdot (X \supset Y)$
2. $T \vee X$
3. $(T \supset \sim Y) \cdot (X \supset \sim W)$
4. $(W \cdot \sim Y) \supset Z$
5. $Y \supset (W \vee A)$ / $Z \vee A$

Sample Solutions to Practice Problems for Test #3

- 1.
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|----------------------------|---------------------------|
| 1. $G \supset H$ | |
| 2. $G \supset I$ | $/ G \supset (H \cdot I)$ |
| 3. G | ACP |
| 4. H | 1, 3, MP |
| 5. I | 2, 3, MP |
| 6. $H \cdot I$ | 4, 5, Conj |
| 7. $G \supset (H \cdot I)$ | 3-6, CP |

QED

- 2.
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|--|----------------|
| 1. $J \equiv K$ | |
| 2. $\sim K \equiv \sim L$ | $/ J \equiv L$ |
| 3. $(J \supset K) \cdot (K \supset J)$ | 1, Equiv |
| 4. $(\sim K \supset \sim L) \cdot (\sim L \supset \sim K)$ | 2, Equiv |
| 5. $J \supset K$ | 3, Simp |
| 6. $\sim L \supset \sim K$ | 4, Com, Simp |
| 7. $K \supset L$ | 6, Cont |
| 8. $J \supset L$ | 5, 7, HS |
| 9. $\sim K \supset \sim L$ | 4, Simp |
| 10. $L \supset K$ | 9, Cont |
| 11. $K \supset J$ | 3, Com, Simp |
| 12. $L \supset J$ | 10, 11, HS |
| 13. $(J \supset L) \cdot (L \supset J)$ | 8, 12, Conj |
| 14. $J \equiv L$ | 13, Equiv |

QED

- 3.
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|---|--|---|
| 1. M | | $/ N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$ |
| 2. N | | ACP |
| 3. O | | ACP |
| 4. P | | ACP |
| 5. $M \cdot N$ | | 1, 2, Conj |
| 6. $O \cdot P$ | | 3, 4, Conj |
| 7. $(M \cdot N) \cdot (O \cdot P)$ | | 5, 6, Conj |
| 8. $P \supset [(M \cdot N) \cdot (O \cdot P)]$ | | 4-7, CP |
| 9. $O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}$ | | 3-8, CP |
| 10. $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$ | | 2-9, CP |

QED

- 4.
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|--|--|----------|
| 1. R | | ACP |
| 2. $R \supset S$ | | ACP |
| 3. S | | 2, 1, MP |
| 4. $(R \supset S) \supset S$ | | 2-3, CP |
| 5. $R \supset [(R \supset S) \supset S]$ | | 1-4, CP |

QED

- 5.
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|----------------------------|--------------|
| 1. $D \supset E$ | |
| 2. $E \supset (F \cdot G)$ | |
| 3. $\sim F \vee \sim G$ | / $\sim D$ |
| 4. D | AIP |
| 5. E | 1, 4, MP |
| 6. $F \cdot G$ | 2, 5, MP |
| 7. F | 6, Simp |
| 8. $\sim G$ | 3, 7, DN, DS |
| 9. G | 6, Com, Simp |
| 10. $G \cdot \sim G$ | 9, 8, Conj |
| 11. $\sim D$ | 4-10, IP |

QED

- 6.
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|---|----------------|
| 1. $(P \vee Q) \vee (\sim R \vee S)$ | |
| 2. $\sim Q \cdot (\sim S \cdot \sim P)$ | / $\sim R$ |
| 3. $\sim Q$ | 2, Simp |
| 4. $(Q \vee P) \vee (\sim R \vee S)$ | 1, Com |
| 5. $Q \vee [P \vee (\sim R \vee S)]$ | 4, Assoc |
| 6. $P \vee (\sim R \vee S)$ | 5, 3, DS |
| 7. $\sim S \cdot \sim P$ | 2, Com, Simp |
| 8. $\sim P$ | 7, Com, Simp |
| 9. $\sim R \vee S$ | 6, 8, DS |
| 10. $\sim S$ | 7, Simp |
| 11. $\sim R$ | 9, 10, Com, DS |

QED

- 7.
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|----------------------------|-------------------------|
| 1. $A \supset B$ | |
| 2. $B \supset D$ | |
| 3. $D \supset A$ | |
| 4. $A \supset \sim D$ | / $\sim A \cdot \sim D$ |
| 5. $A \supset D$ | 1, 2, HS |
| 6. $\sim D \supset \sim A$ | 5, Cont |
| 7. $A \supset \sim A$ | 4, 6, HS |
| 8. $\sim A \vee \sim A$ | 7, Impl |
| 9. $\sim A$ | 8, Taut |
| 10. $\sim D$ | 3, 9, MT |
| 11. $\sim A \cdot \sim D$ | 9, 10, Conj |

QED

- 8.
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|--|--------------|
| 1. $\sim \{A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]\}$ | AIP |
| 2. $\sim A \cdot \sim [(\sim A \vee B) \cdot (\sim A \vee C)]$ | 1, DM |
| 3. $\sim A$ | 2, Simp |
| 4. $\sim [(\sim A \vee B) \cdot (\sim A \vee C)]$ | 2, Com, Simp |
| 5. $\sim [\sim A \vee (B \cdot C)]$ | 4, Dist |
| 6. $A \cdot \sim (B \cdot C)$ | 5, DM, DN |
| 7. A | 6, Simp |
| 8. $A \cdot \sim A$ | 7, 3, Conj |
| 9. $A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$ | 1-8, IP, DN |

QED

- 9.
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|---|---------------------|
| 1. $(A \vee B) \supset \sim(F \cdot D)$ | |
| 2. $\sim(A \cdot \sim D)$ | |
| 3. $\sim F \supset \sim(C \cdot D)$ | |
| 4. $C \vee A$ | / $A \equiv \sim C$ |
| 5. A | ACP |
| 6. $A \vee B$ | 5, Add |
| 7. $\sim(F \cdot D)$ | 6, 1, 6, MP |
| 8. $\sim F \vee \sim D$ | 7, DM |
| 9. $\sim A \vee \sim \sim D$ | 2, DM |
| 10. $\sim \sim D$ | 9, 5, DM, DS |
| 11. $\sim F$ | 8, 10, Com, DS |
| 12. $\sim(C \cdot D)$ | 3, 11, MP |
| 13. $\sim C \vee \sim D$ | 12, DM |
| 14. $\sim C$ | 13, 10, Com, DS |
| 15. $A \supset \sim C$ | 5-14, CP |
| 16. $\sim C \supset A$ | 4, DN, Impl |
| 17. $(A \supset \sim C) \cdot (\sim C \supset A)$ | 15, 16, Conj |
| 18. $A \equiv \sim C$ | 17, Equiv |

QED

- 10.
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|----------------------------|---------------|
| 1. $(A \cdot B) \supset C$ | |
| 2. $(F \cdot D) \supset E$ | |
| 3. $A \cdot D$ | |
| 4. $B \vee F$ | / $C \vee E$ |
| 5. $\sim(C \vee E)$ | AIP |
| 6. $\sim C \cdot \sim E$ | 5, DM |
| 7. $\sim C$ | 6, Simp |
| 8. $\sim(A \cdot B)$ | 1, 7, MT |
| 9. $\sim A \vee \sim B$ | 8, DM |
| 10. A | 3, Simp |
| 11. $\sim B$ | 9, 10, DN, DS |
| 12. F | 4, 11, DS |
| 13. D | 3, Com, Simp |
| 14. $F \cdot D$ | 12, 13, Conj |
| 15. E | 2, 14, MP |
| 16. $\sim E$ | 6, Com, Simp |
| 17. $E \cdot \sim E$ | 15, 16, Conj |
| 18. $C \vee E$ | 5-17, IP, DN |

QED

11.	1. $\sim F \supset (G \cdot H)$	
	2. $F \supset (I \vee J)$	
	3. $\sim [G \cdot (K \cdot \sim J)]$	
	4. $K \equiv (L \cdot \sim J)$	$/ K \supset I$
	5. K	ACP
	6. $[K \supset (L \cdot \sim J)] \cdot [(L \cdot \sim J) \supset K]$	4, Equiv
	7. $K \supset (L \cdot \sim J)$	6, Simp
	8. $L \cdot \sim J$	7, 5, MP
	9. $\sim J$	8, Com, Simp
	10. $K \cdot \sim J$	5, 9, Conj
	11. $\sim G \vee \sim (K \cdot \sim J)$	3, DM
	12. $\sim (K \cdot \sim J) \vee \sim G$	11, Com
	13. $\sim G$	12, 10, DN, DS
	14. $\sim I$	AIP
	15. $\sim I \cdot \sim J$	14, 9, Conj
	16. $\sim (I \vee J)$	15, DM
	17. $\sim F$	2, 16, MT
	18. $G \cdot H$	1, 17, MP
	19. G	18, Simp
	20. $G \cdot \sim G$	19, 13, Conj
	21. I	14-20, IP
	22. $K \supset I$	5-21, CP

QED

12.	1. $(I \cdot E) \supset \sim F$	
	2. $F \vee (G \cdot H)$	
	3. $I \equiv E$	$/ I \supset G$
	4. $(I \supset E) \cdot (E \supset I)$	3, Equiv
	5. I	ACP
	6. $I \supset E$	4, Simp
	7. E	6, 5, MP
	8. $I \cdot E$	5, 7, Conj
	9. $\sim F$	1, 8, MP
	10. $G \cdot H$	2, 9, DS
	11. G	10, Simp
	12. $I \supset G$	5-11, CP

13.	1. $P \cdot Q$	ACP
	2. P	1, Simp
	3. $P \vee R$	2, Add
	4. Q	1, Com, Simp
	5. $Q \vee R$	4, Add
	6. $(P \vee R) \cdot (Q \vee R)$	3, 5, Conj
	7. $(P \cdot Q) \supset [(P \vee R) \cdot (Q \vee R)]$	1-6, IP

QED

14.	1. $\neg[(A \supset B) \vee (\neg A \supset C)]$	AIP
	2. $\neg(A \supset B) \bullet \neg(\neg A \supset C)$	1, DM
	3. $\neg(A \supset B)$	2, Simp
	4. $\neg(\neg A \vee B)$	3, Impl
	5. $A \bullet \neg B$	4, DM, DN
	6. $\neg(\neg A \supset C)$	2, Com, Simp
	7. $\neg(A \vee C)$	6, Impl, DN
	8. $\neg A \bullet \neg C$	8, DM
	9. A	5, Simp
	10. $\neg A$	8, Simp
	11. $A \bullet \neg A$	9, 10, Conj
	12. $(A \supset B) \vee (\neg A \supset C)$	1-11, IP, DN

QED

15.	1. $(T \supset W) \bullet (X \supset Y)$	
	2. $T \vee X$	
	3. $(T \supset \neg Y) \bullet (X \supset \neg W)$	
	4. $(W \bullet \neg Y) \supset Z$	
	5. $Y \supset (W \vee A)$	/ $Z \vee A$
	6. $W \vee Y$	1, 2, CD
	7. $\neg Y \vee \neg W$	3, 2, CD
	8. $\neg(Z \vee A)$	AIP
	9. $\neg Z \bullet \neg A$	8, DM
	10. $\neg Z$	9, Simp
	11. $\neg(W \bullet \neg Y)$	4, 10, MT
	12. $\neg W \vee Y$	11, DM, DN
	13. $\neg W \vee \neg Y$	7, Com
	14. $(\neg W \vee Y) \bullet (\neg W \vee \neg Y)$	12, 13, Conj
	15. $\neg W \vee (Y \bullet \neg Y)$	14, Dist
	16. $Y \bullet \neg Y$	AIP
	17. $\neg(Y \bullet \neg Y)$	16, IP
	18. $\neg W$	15, 17, Com, DS
	19. Y	6, 18, DS
	20. $W \vee A$	5, 19, MP
	21. A	20, 18, DS
	22. $\neg A$	9, Com, Simp
	23. $A \bullet \neg A$	21, 22, Conj
	24. $Z \vee A$	8-23, IP, DN

QED